

I B.Tech Supplementary Examinations, Aug/Sep 2008
MATHEMATICS-I

(Common to Civil Engineering, Electrical & Electronic Engineering,
 Mechanical Engineering, Electronics & Communication Engineering,
 Computer Science & Engineering, Chemical Engineering, Electronics &
 Instrumentation Engineering, Bio-Medical Engineering, Information
 Technology, Electronics & Control Engineering, Mechatronics, Computer
 Science & Systems Engineering, Electronics & Telematics, Metallurgy &
 Material Technology, Electronics & Computer Engineering, Production
 Engineering, Aeronautical Engineering, Instrumentation & Control
 Engineering, Bio-Technology and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Solve $y(2x^2y + e^x) dx = (e^x + y^3) dy$.
 (b) Suppose that an object is heated to 300° F and allowed to cool in a room whose air temperature is 80° F, if after 10 minutes the temperature of the object is 250° F. What will be its temperature after 20 minutes? [8+8]
2. (a) Solve $(D^2 + 3D + 2)y = \sin 3x$
 (b) Solve $(D^2 - 4D + 3)y = \cos 2x$. [8+8]
3. (a) Verify Lagrange's mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in $[0,4]$.
 (b) Find the maximum and minimum values of the function $f(x) = x^5 - 3x^4 + 5$. [8+8]
4. (a) Find the centre of curvature of $x = a \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$, $y = a \sec \theta$.
 (b) Find the evolute of the parabola $y^2 = 4ax$, considering it as the envelope of normals. [8+8]
5. (a) By changing into polar coordinates evaluate the integral $\int_0^{\infty} \int_0^{\infty} \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}} dx dy$.
 (b) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy$. [8+8]
6. (a) Discuss the convergence of $\frac{x}{1.3} + \frac{x^2}{5.7} + \frac{x^3}{7.9} + \dots$, ($x > 0$)
 (b) Test the convergence of $\sum n^p / n!$ [8+8]
7. (a) Find the work done in moving a particle by the force $\vec{F} = 3x^2 \vec{i} + (2xz - y)\vec{j} + z \vec{k}$ along the line joining $(0,0,0)$ to $(2,1,3)$

Code No: R07A1BS02

Set No. 1

- (b) Using Green's theorem evaluate $\int_C (2xy - x^2) dx + (x^2 + y^2) dy$ where C is the closed curve of the region bounded $y = x^2$ and $y^2 = x$. [8+8]
8. (a) Find $L^{-1} [(1 + e^{-\pi s}) / (s^2 + 1)]$.
- (b) Using convolution theorem find $L^{-1} [1 / (s^2 - 1)(s^2 + 25)]$. [8+8]

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1. (a) Show that $(5x^4 + 3x^2y^2 - 2xy^3) dx + (2x^3y - 3x^2y^2 - 5y^4) dy = 0$.
 (b) The world population at the beginning of 1970 was 3.6 billion. The weight of the earth is 6.586×10^{21} tons. If the population continues to increase exponentially, with a growth constant $k = 0.02$ and with time measured in years, in what year will the weight of all people equal the weight of the earth, if we assume that the average person weighs 120 pounds? [8+8]
2. (a) Solve $(D^2 + 2D + 2)y = e^{-x} + \sin 2x$
 (b) Solve the equation $(D^2 - 2D + 2)y = e^x \tan x$. [8+8]
3. (a) Explain why Mean Value Theorem does not hold for $f(x) = x^{\frac{2}{3}}$.
 (b) If $x = e^r \sec \theta$, $y = e^r \tan \theta$ prove that $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$. [8+8]
4. (a) Find the radius of curvature of $x = \log t$, $y = \frac{1}{2}(t + t^{-1})$ at $t = 1$.
 (b) Find the envelope of $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ where ' θ ' is a parameter. [8+8]
5. (a) Evaluate $\iint (x + y) dx dy$, over the region in the positive quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
 (b) Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r (b - a)$ when $b > a$ and hence deduce the surface area of the sphere. [8+8]
6. (a) Discuss the convergence of $\sum (n^2 + n + 1)x^n / (n + 2)!$, ($x > 0$)
 (b) Examine the convergence of $\frac{1}{1.2.3} - \frac{5}{3.4.5} + \frac{9}{5.6.7} - \frac{13}{7.8.9} + \dots$ [10+6]
7. (a) If \vec{r} is the position vector of the point (x, y, z) , prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$.

- (b) Use Gauss divergence theorem to evaluate $\iint_S (yz^2\vec{i} + zx^2\vec{j} + 2z^2\vec{k}) \cdot d\vec{S}$, where S is the closed surface bounded by the xy-plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane. [8+8]
8. (a) Find $L^{-1} [(1 + e^{-\pi s}) / (s^2 + 1)]$.
- (b) Using convolution theorem find $L^{-1} [1 / (s^2 - 1)(s^2 + 25)]$. [8+8]

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1. (a) Solve $(3y + 2x + 4) dx - (4x + 6y + 5) dy = 0$.
 (b) Bacteria in a culture grows exponentially so that the initial number has doubled in three hours. How many times the initial number will be present after 9 hours. [8+8]
2. Solve $(D^2 - 4D + 4) y = e^{2x} + \cos 2x + e^x \sin 2x$. [16]
3. (a) Verify generalised mean value theorem for $f(x) = e^x$, $g(x) = e^{-x}$ in $[3, 7]$ and find the value of 'c'.
 (b) If the strength of a beam of a rectangular cross section varies as the product of its breadth and square of its depth, find the dimensions of the strongest beam that can be cut from a circular log of radius R. [8+8]
4. (a) Find the position on $y^2 = 8x$ at which the curvature is $\frac{16}{125}$.
 (b) Given that $x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$ is the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ find the necessary relation between a and b. [8+8]
5. (a) By changing the order of integration, evaluate $\int_0^1 \int_1^{2-x} xy \, dx dy$.
 (b) Find the length of the curve $3x^2 = y^3$ between $y = 0$ and $y = 1$. [8+8]
6. (a) Examine the convergence of
$$\sum \left[\frac{1.4.7 \dots (3n-2)}{3.6.9 \dots 3n} \right]^2$$

 (b) Examine the convergence of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ [8+8]
7. Verify Green's theorem in the plane for $\int_C (x^2 - xy^3) dx + (y^2 - 2xy) dy$ where C is a square with vertices (0,0), (2,0), (2,2), (0,2) [16]

8. (a) Using Laplace transform evaluate $\int_0^{\infty} t e^{-t} \sin t \, dt$.

(b) Using Laplace transform, solve $(D^2 + 4D + 5)y = 5$, given that $y(0) = 0, y''(0) = 0$.

[8+8]

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1. (a) Solve $2xy \, dy - (x^2 + y^2 + 1) \, dx = 0$.
(b) Find the equation of Orthogonal trajectories of circles $r = a \cos\theta$. [8+8]
2. Solve $(D^2 + 4D + 3)y = e^x \cos 2x - \cos 3x - 3x^3$. [16]
3. (a) Prove that if $x > 0$, $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$.
(b) Investigate the maxima and minima, if any, of the function
 $f(x) = x^3 y^2 (1 - x - y)$. [8+8]
4. (a) Find the radius of curvature of $\sqrt{a} = \sqrt{r} \cos \frac{\theta}{2}$ at (r, θ) .
(b) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ where $a^2 + b^2 = 4$. [8+8]
5. (a) Evaluate $\iint (x+y) \, dx \, dy$, over the region in the positive quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
(b) Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r (b - a)$ when $b > a$ and hence deduce the surface area of the sphere. [8+8]
6. (a) Examine the convergence of
$$\sum \frac{2^2 \cdot 4^2 \cdot 6^2 \dots (2n)^2}{3^2 \cdot 4^2 \cdot 5^2 \dots (2n+2)^2}$$

(b) Examine the convergence of
$$\sum (n^3 - 5n^2 + 7) / (n^5 + 4n^4 - n)$$
 [8+8]
7. Verify Gauss divergence theorem for $\vec{F} = x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}$ taken over the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$. [16]
8. (a) Find $L^{-1} [s^2 / (s^2 + 4)(s^2 + 25)]$.
(b) Find $L^{-1} [2 + s \log(s-1) / (s+1)]$. [8+8]
